

Black Box Software Testing

(Academic Course - Fall 2001)

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Section: 23 :

Combination Testing

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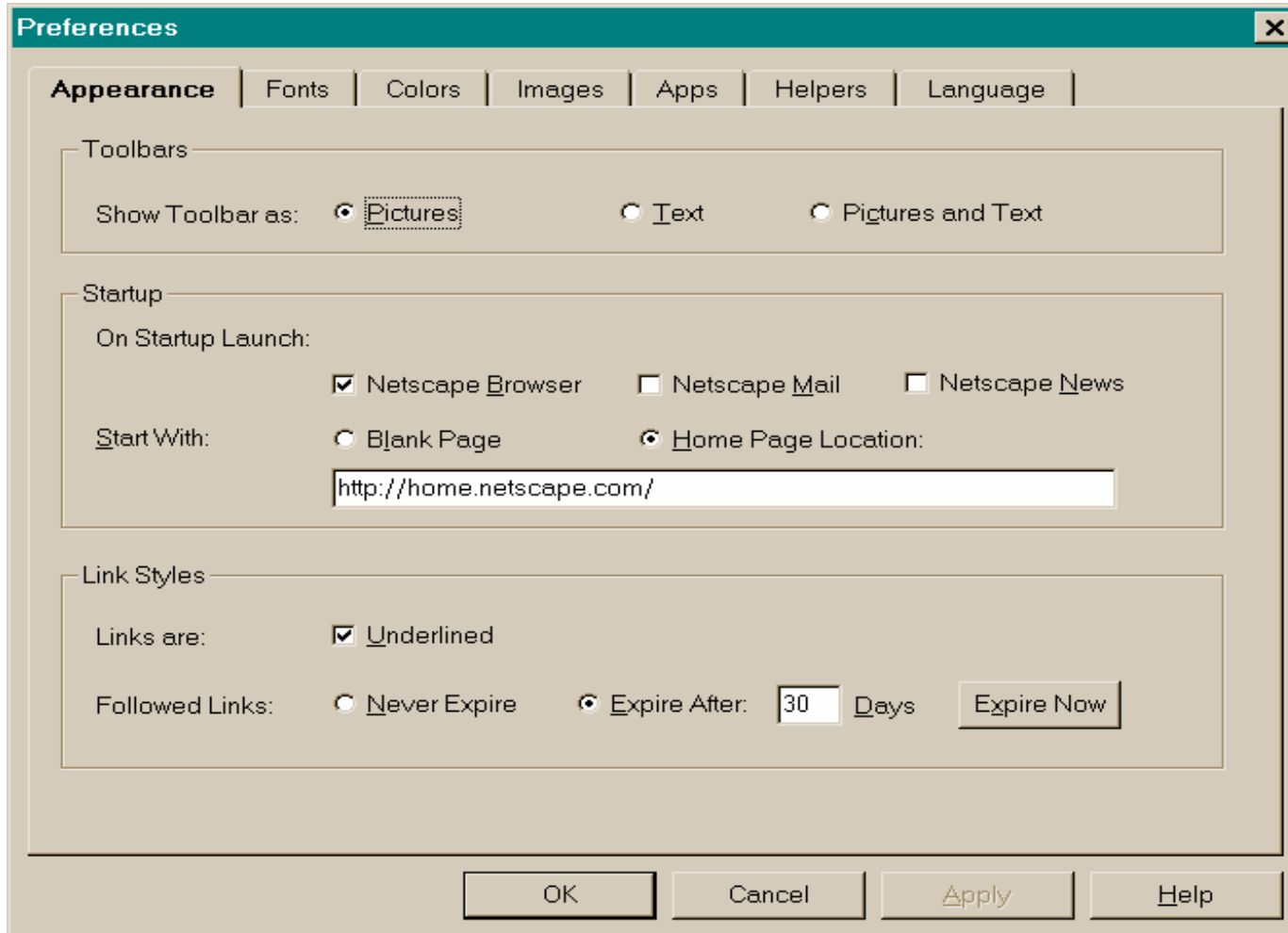
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Combination Chart

| | Var 1 | Var 2 | Var 3 | Var 4 | Var 5 |
|--------|----------|----------|----------|----------|----------|
| Test 1 | Value 11 | Value 12 | Value 13 | Value 14 | Value 15 |
| Test 2 | Value 21 | Value 22 | Value 23 | Value 24 | Value 25 |
| Test 3 | Value 31 | Value 32 | Value 33 | Value 34 | Value 35 |
| Test 4 | Value 41 | Value 42 | Value 43 | Value 44 | Value 45 |
| Test 5 | Value 51 | Value 52 | Value 53 | Value 54 | Value 55 |
| Test 6 | Value 61 | Value 62 | Value 63 | Value 64 | Value 65 |

Testing Variables in Combination



*The
Netscape
Preferences
dialog:*

Testing Variables in Combination

- If we just look at the Appearance tab of the Netscape Preferences dialog, we see the following variables:
 - Toolbars -- 3 choices (P, T, B)
(**pictures**, **text** or **both**)
 - On Startup Launch -- 3 choices (B, M, N)
(**browser**, **mail**, **news**). Each of these is an independent binary choice.
 - Start With -- 3 choices (B,V,E)
(**blank** page, home page names a **valid** file, home page name has a **syntax error**)
(Many more cases are possible, but let's keep this simple and ignore that for a few slides)
 - Links -- 2 choices (D,U)
(**don't** underline, **underlined**)
 - Followed Links -- 2 choices (N,E)
(**never** expire, **expire** after 30 days) (Many more cases are possible)

Testing Variables in Combination

- I simplified the combinations by simplifying the choices for two of the fields.
- In the Start With field, I used either a valid home page name or a blank name. Some other test cases that could go into this field are:
 - file name (name.htm instead of using http:// to define a protocol) on the local drive, the local network drive, or the remote drive
 - maximum length file names, maximum length paths
 - invalid file names and paths
- For combination testing, select a few of these that look like they might interact with other variables. Test the rest independently.
- Similarly for the Expire After field. This lets you enter the number of days to store links. If you use more than one value, use boundary cases, not all the numbers in the range.
- *In multi-variable testing, use partition analysis or other special values instead of testing all values in combination with all other variables' all values.*

Testing Variables in Combination

•We can create $3 \times 2 \times 2 \times 2 \times 3 \times 2 \times 2 = 288$ different test cases by testing these variables in combination. Here are some examples, from a combination table.

| | Toolbars PTB | On Startup, Browser Y/N | On Startup, Mail Y/N | On Startup, News Y/N | Start With BVE | Links DU | Followed NE |
|----------------|--------------|-------------------------|----------------------|----------------------|----------------|----------|-------------|
| Test #1 | P | Y | Y | Y | B | D | N |
| 2 | P | Y | Y | N | B | D | E |
| 3 | P | Y | N | Y | B | U | N |
| 4 | P | Y | N | N | B | U | E |
| 5 | P | Y | Y | Y | V | D | N |
| 6 | P | Y | Y | N | V | D | E |
| 7 | P | N | N | Y | V | U | N |
| 8 | P | N | N | N | V | U | E |
| 9 | P | N | Y | Y | E | D | N |
| 10 | P | N | Y | N | E | D | E |
| 11 | P | N | N | Y | E | U | N |
| 12 | P | N | N | N | E | U | E |

Here are the 288 test cases. Every value of every variable is combined with every combination of the other variables.

| | | | | | | |
|----|---------------|---------------|---------------|---------------|---------------|---------------|
| 1 | P Y Y Y B D N | P N Y Y B D N | T Y Y Y B D N | T N Y Y B D N | B Y Y Y B D N | B N Y Y B D N |
| 2 | P Y Y Y V D E | P N Y Y V D E | T Y Y Y V D E | T N Y Y V D E | B Y Y Y V D E | B N Y Y V D E |
| 3 | P Y Y Y E D N | P N Y Y E D N | T Y Y Y E D N | T N Y Y E D N | B Y Y Y E D N | B N Y Y E D N |
| 4 | P Y Y Y B U E | P N Y Y B U E | T Y Y Y B U E | T N Y Y B U E | B Y Y Y B U E | B N Y Y B U E |
| 5 | P Y Y Y V U N | P N Y Y V U N | T Y Y Y V U N | T N Y Y V U N | B Y Y Y V U N | B N Y Y V U N |
| 6 | P Y Y Y E U E | P N Y Y E U E | T Y Y Y E U E | T N Y Y E U E | B Y Y Y E U E | B N Y Y E U E |
| 7 | P Y Y N B D N | P N Y N B D N | T Y Y N B D N | T N Y N B D N | B Y Y N B D N | B N Y N B D N |
| 8 | P Y Y N V D E | P N Y N V D E | T Y Y N V D E | T N Y N V D E | B Y Y N V D E | B N Y N V D E |
| 9 | P Y Y N E D N | P N Y N E D N | T Y Y N E D N | T N Y N E D N | B Y Y N E D N | B N Y N E D N |
| 10 | P Y Y N B U E | P N Y N B U E | T Y Y N B U E | T N Y N B U E | B Y Y N B U E | B N Y N B U E |
| 11 | P Y Y N V U N | P N Y N V U N | T Y Y N V U N | T N Y N V U N | B Y Y N V U N | B N Y N V U N |
| 12 | P Y Y N E U E | P N Y N E U E | T Y Y N E U E | T N Y N E U E | B Y Y N E U E | B N Y N E U E |
| 13 | P Y Y Y B D E | P N Y Y B D E | T Y Y Y B D E | T N Y Y B D E | B Y Y Y B D E | B N Y Y B D E |
| 14 | P Y Y Y V D N | P N Y Y V D N | T Y Y Y V D N | T N Y Y V D N | B Y Y Y V D N | B N Y Y V D N |
| 15 | P Y Y Y E D E | P N Y Y E D E | T Y Y Y E D E | T N Y Y E D E | B Y Y Y E D E | B N Y Y E D E |
| 16 | P Y Y Y B U N | P N Y Y B U N | T Y Y Y B U N | T N Y Y B U N | B Y Y Y B U N | B N Y Y B U N |
| 17 | P Y Y Y V U E | P N Y Y V U E | T Y Y Y V U E | T N Y Y V U E | B Y Y Y V U E | B N Y Y V U E |
| 18 | P Y Y Y E U N | P N Y Y E U N | T Y Y Y E U N | T N Y Y E U N | B Y Y Y E U N | B N Y Y E U N |
| 19 | P Y Y N B D E | P N Y N B D E | T Y Y N B D E | T N Y N B D E | B Y Y N B D E | B N Y N B D E |
| 20 | P Y Y N V D N | P N Y N V D N | T Y Y N V D N | T N Y N V D N | B Y Y N V D N | B N Y N V D N |
| 21 | P Y Y N E D E | P N Y N E D E | T Y Y N E D E | T N Y N E D E | B Y Y N E D E | B N Y N E D E |
| 22 | P Y Y N B U N | P N Y N B U N | T Y Y N B U N | T N Y N B U N | B Y Y N B U N | B N Y N B U N |
| 23 | P Y Y N V U E | P N Y N V U E | T Y Y N V U E | T N Y N V U E | B Y Y N V U E | B N Y N V U E |
| 24 | P Y Y N E U N | P N Y N E U N | T Y Y N E U N | T N Y N E U N | B Y Y N E U N | B N Y N E U N |
| 25 | P Y N Y B D E | P N N Y B D E | T Y N Y B D E | T N N Y B D E | B Y N Y B D E | B N N Y B D E |
| 26 | P Y N Y V D N | P N N Y V D N | T Y N Y V D N | T N N Y V D N | B Y N Y V D N | B N N Y V D N |
| 27 | P Y N Y E D E | P N N Y E D E | T Y N Y E D E | T N N Y E D E | B Y N Y E D E | B N N Y E D E |
| 28 | P Y N Y B U N | P N N Y B U N | T Y N Y B U N | T N N Y B U N | B Y N Y B U N | B N N Y B U N |
| 29 | P Y N Y V U E | P N N Y V U E | T Y N Y V U E | T N N Y V U E | B Y N Y V U E | B N N Y V U E |
| 30 | P Y N Y E U N | P N N Y E U N | T Y N Y E U N | T N N Y E U N | B Y N Y E U N | B N N Y E U N |
| 31 | P Y N N B D E | P N N N B D E | T Y N N B D E | T N N N B D E | B Y N N B D E | B N N N B D E |
| 32 | P Y N N V D N | P N N N V D N | T Y N N V D N | T N N N V D N | B Y N N V D N | B N N N V D N |
| 33 | P Y N N E D E | P N N N E D E | T Y N N E D E | T N N N E D E | B Y N N E D E | B N N N E D E |
| 34 | P Y N N B U N | P N N N B U N | T Y N N B U N | T N N N B U N | B Y N N B U N | B N N N B U N |
| 35 | P Y N N V U E | P N N N V U E | T Y N N V U E | T N N N V U E | B Y N N V U E | B N N N V U E |
| 36 | P Y N N E U N | P N N N E U N | T Y N N E U N | T N N N E U N | B Y N N E U N | B N N N E U N |
| 37 | P Y N Y B D N | P N N Y B D N | T Y N Y B D N | T N N Y B D N | B Y N Y B D N | B N N Y B D N |
| 38 | P Y N Y V D E | P N N Y V D E | T Y N Y V D E | T N N Y V D E | B Y N Y V D E | B N N Y V D E |
| 39 | P Y N Y E D N | P N N Y E D N | T Y N Y E D N | T N N Y E D N | B Y N Y E D N | B N N Y E D N |
| 40 | P Y N Y B U E | P N N Y B U E | T Y N Y B U E | T N N Y B U E | B Y N Y B U E | B N N Y B U E |
| 41 | P Y N Y V U N | P N N Y V U N | T Y N Y V U N | T N N Y V U N | B Y N Y V U N | B N N Y V U N |
| 42 | P Y N Y E U E | P N N Y E U E | T Y N Y E U E | T N N Y E U E | B Y N Y E U E | B N N Y E U E |
| 43 | P Y N N B D N | P N N N B D N | T Y N N B D N | T N N N B D N | B Y N N B D N | B N N N B D N |
| 44 | P Y N N V D E | P N N N V D E | T Y N N V D E | T N N N V D E | B Y N N V D E | B N N N V D E |
| 45 | P Y N N E D N | P N N N E D N | T Y N N E D N | T N N N E D N | B Y N N E D N | B N N N E D N |
| 46 | P Y N N B U E | P N N N B U E | T Y N N B U E | T N N N B U E | B Y N N B U E | B N N N B U E |
| 47 | P Y N N V U N | P N N N V U N | T Y N N V U N | T N N N V U N | B Y N N V U N | B N N N V U N |
| 48 | P Y N N E U E | P N N N E U E | T Y N N E U E | T N N N E U E | B Y N N E U E | B N N N E U E |

Testing Variables in Combination

- To simplify this, many testers would test variables in pairs.
- That can be useful if you understand specific relationships between the variables, but if you are doing general combination testing, then restricting your attention to pairs is less efficient and less simple than you might expect.
- Look at all the pairs you'd have to test, if you tested them all, pair by pair -- 109 of them. This is better than 288, but not much.

Testing Variables in Combination

| | Toolbars P/T/B | Browser Y/N | Mail Y/N | News Y/N | Start B/N/E | Links D/U | Followed N/E | TOTAL PAIRS |
|-----------------------|-------------------|----------------|-------------|-------------|----------------|--------------|-----------------|----------------|
| Toolbars 3 choices | ----- | 6 | 6 | 6 | 9 | 6 | 6 | 39 |
| Browser 2 choices | ----- | ----- | 4 | 4 | 6 | 4 | 4 | 22 |
| Mail 2 choices | ----- | ----- | ----- | 4 | 6 | 4 | 4 | 18 |
| News 2 choices | ----- | ----- | ----- | ----- | 6 | 4 | 4 | 14 |
| Start 3 choices | ----- | ----- | ----- | ----- | ----- | 6 | 6 | 12 |
| Links 2 choices | ----- | ----- | ----- | ----- | ----- | ----- | 4 | 4 ----- |
| Followed 2 choices | ----- | ----- | ----- | ----- | ----- | ----- | ----- | <i>109</i> |

Testing Variables in Combination

- Now consider testing every possible pair, but testing many pairs simultaneously.
- We are creating test cases that combine all variables at once, and that assure that every value of every variable is paired with every other value of every other variable.
- Each of these test cases covers 21 pairs. In general, each test case that combines N variables includes $N(N-1) / 2$ pairs

| | Toolbars PTB | On Startup, Browser Y/N | On Startup, Mail Y/N | On Startup, News Y/N | Start With BVE | Links DU | Followed NE |
|----------------|--------------|-------------------------|----------------------|----------------------|----------------|----------|-------------|
| Test #1 | P | Y | Y | Y | B | D | N |
| 2 | P | Y | N | N | V | D | E |
| 3 | P | N | Y | Y | E | U | E |
| 4 | T | Y | N | Y | E | U | N |
| 5 | T | N | Y | N | B | D | E |
| 6 | T | N | N | Y | V | D | N |
| 7 | B | Y | N | N | B | U | E |
| 8 | B | N | Y | Y | V | U | E |
| 9 | B | N | N | N | E | D | N |

Combinations Exercise



- Here is a simple Find dialog. It takes three inputs:
 - Find what: a text string
 - Match case: yes or no
 - Direction: up or down
- Simplify this by considering only three values for the text string, “lowercase” and “Mixed Cases” and “CAPITALS”.

The AETG System: An Approach to Testing Based on Combinatorial Design

Appeared in July 1997 issue of IEEE Transactions On Software Engineering (Vol. 23, No. 7)

By

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M. L. Fredman - Rutgers University
G. C. Patton - Bellcore

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Abstract

This paper describes a new approach to testing that uses combinatorial designs to generate tests that cover the pair-wise, triple or n-way combinations of a system's test parameters. These are the parameters that determine the system's test scenarios. Examples are system configuration parameters, user inputs and other external events. We implemented this new method in the AETG

Combinations Exercise

- Find has three cases:
 - L M C (lower, mixed, capitals)
- Match has two cases:
 - Y N (yes, no)
- Direction has two cases:
 - U D (up, down)

• 1. Total cases is $3 \times 2 \times 2 = 12$

• 2. Full set has 12 tests

- L Y U M Y U C Y U
- L Y D M Y D C Y D
- L N U M N U C N U
- L N D M N D C N D

3. A reduced set is

L Y U

L N D

M Y D

M N U

C Y U

C N D

4. The total is 6

Combination Testing

- **Imagine a program with 3 variables, V1 has 3 possible values, V2 has 2 possible values and V3 has 2 possible values.**
- **If V1 and V2 and V3 are independent, the number of possible combinations is 12 (3 x 2 x 2)**
- **Building a simple combination table:**
 - Label the columns with the variable names, listing variables in descending order (of number of possible values)
 - Each column (before the last) will have repetition. Suppose that A, B, and C are in column K of N columns. To determine how many times (rows in which) to repeat A before creating a row for B, multiply the number of variable values in columns K+1, K+2, . . . , N.

Combination Testing

- Building an all-pairs combination table:
 - Label the columns with the variable names, listing variables in descending order (of number of possible values)
 - If the variable in column 1 has $V1$ possible values and the variable in column 2 has $V2$ possible values, then there will be at least $V1 \times V2$ rows (draw the table this way but leave a blank row or two between repetition groups in column 1).
 - Fill in the table, one column at a time. The first column repeats each of its elements $V2$ times, skips a line, and then starts the repetition of the next element. For example, if variable 1's possible values are A, B, C and $V2$ is 2, then column 1 would contain A, A, blank row, B, B, blank row, C, C, blank row.

Combination Testing

- **Building an all-pairs combination table:**
 - In the second column, list all the values of the variable, skip the line, list the values, etc. For example, if variable 2's possible values are X,Y, then the table looks like this so far

| | |
|---|---|
| A | X |
| A | Y |
| | |
| B | X |
| B | Y |
| | |
| C | X |
| C | Y |
| | |

Combination Testing

- Building an all-pairs combination table:
 - Each section of the third column (think of AA as defining a section, BB as defining another, etc.) will have to contain every value of variable 3. Order the values such that the variables also make all pairs with variable 2.
 - Suppose variable 2 can be 1,0
 - The third section can be filled in either way, and you might highlight it so that you can reverse it later. The decision (say 1,0) is arbitrary.
- *Now that we've solved the 3-column exercise, let's try adding more variables. Each of them will have two values.*

| | | |
|---|---|---|
| A | X | 1 |
| A | Y | 0 |
| | | |
| B | X | 0 |
| B | Y | 1 |
| | | |
| C | X | |
| C | Y | |
| | | |

Combination Testing

- The 4th column went in easily (note that we started by making sure we hit all pairs of values of column 4 and column 2, then all pairs of column 4 and column 3).
- Watch this first attempt on column 5. We achieve all pairs of GH with columns 1, 2, and 3, but miss it for column 4.
- The most recent arbitrary choice was HG in the 2nd section. (Once that was determined, we picked HG for the third in order to pair H with a 1 in the third column.)
- So we will erase the last choice and try again:

| | | | | |
|---|---|---|---|---|
| A | X | 1 | E | G |
| A | Y | 0 | F | H |
| | | | | |
| B | X | 0 | F | H |
| B | Y | 1 | E | G |
| | | | | |
| C | X | 1 | F | H |
| C | Y | 0 | E | G |
| | | | | |

Combination Testing

- We flipped the last arbitrary choice (column 5, section 2, to GH from HG) and erased section 3. We then fill in section 3 by checking for missing pairs. GH, GH gives us two XG, XG pairs, so we flip to HP for the third section and have a column 2 X with a column 5 H and a column 2 Y with a column 5 G as needed to obtain all pairs.

| | | | | |
|---|---|---|---|---|
| A | X | 1 | E | G |
| A | Y | 0 | F | H |
| | | | | |
| B | X | 0 | F | G |
| B | Y | 1 | E | H |
| | | | | |
| C | X | 1 | F | H |
| C | Y | 0 | E | G |
| | | | | |

Combination Testing

- But when we add the next column, we see that we just can't achieve all pairs with 6 values. The first one works up to column 4 but then fails to get pair EJ or FI. The next fails on GJ, HI

| | | | | | |
|---|---|---|---|---|---|
| A | X | 1 | E | G | I |
| A | Y | 0 | F | H | J |
| | | | | | |
| B | X | 0 | F | G | J |
| B | Y | 1 | E | H | I |
| | | | | | |
| C | X | 1 | F | H | J |
| C | Y | 0 | E | G | I |
| | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| A | X | 1 | E | G | I |
| A | Y | 0 | F | H | J |
| | | | | | |
| B | X | 0 | F | G | I |
| B | Y | 1 | E | H | J |
| | | | | | |
| C | X | 1 | F | H | J |
| C | Y | 0 | E | G | I |
| | | | | | |

Combination Testing

- When all else fails, add rows. We need one for GJ and one for HI, so add two rows. In general, we would need as many rows as the last column has values.
- The other values in the two rows are arbitrary, leave them blank and fill them in as needed when you add new columns. At the very end, fill the remaining blank ones with arbitrary values

| | | | | | |
|---|---|---|---|---|---|
| A | X | 1 | E | G | I |
| A | Y | 0 | F | H | J |
| | | | | G | J |
| B | X | 0 | F | G | I |
| B | Y | 1 | E | H | J |
| | | | | H | I |
| C | X | 1 | F | H | J |
| C | Y | 0 | E | G | I |
| | | | | | |

Combination Testing

- If a variable is continuous but maps to a number line, partition and use boundaries as the distinct values under test. If all variables are continuous, we end up with all pairs of all boundary tests of all variables. We don't achieve all triples, all quadruples, etc.
- If some combinations are of independent interest, add them to the list of n-tuples to test.
 - With the six columns of the example, we reduced 96 tests to 8. Give a few back (make it 12 or 15 tests) and you still get enormous reduction.
 - Examples of “independent interest” are known (from tech support) high risk cases, cases that jointly stress memory, configuration combinations (Var 1 is operating systems, Var 2 is printers, etc.) that are prevalent in the market, etc.

Combination: Interesting Reading

- Cohen, Dalal, Parelus, Patton, “The Combinatorial Design Approach to Automatic Test Generation”, IEEE Software, Sept. 96
<http://www.argreenhouse.com/papers/gcp/AETGissre96.shtml>
- Cohen, Dalal, Fredman, Patton, “The AETG System: An Approach to Testing Based on Combinatorial Design”, IEEE Trans on SW Eng. Vol 23#7, July 97
<http://www.argreenhouse.com/papers/gcp/AETGiieee97.shtml>
- **Several other papers on AETG are available at**
<http://aetgweb.argreenhouse.com/papers.html>
- Also interesting: a discussion of orthogonal arrays
<http://www.stsc.hill.af.mil/CrossTalk/1997/oct/planning.html>

Combinations Exercise

- 1 How many combinations of these three variables are possible?
- 2 List ALL the combinations of these three variables.
- 3 Now create combination tests that cover all possible pairs of values, but don't try to cover all possible triplets. List one such set.
- 4 How many test cases are in this set?